MOTION IN APLANE

INTRODUCTION :

A study of motion will involve the introduction of a variety of quantities that are used to describe the physical world. Examples of such quantities include distance, displacement, speed, velocity, acceleration, force, mass, momentum, energy, work, power, etc. All these quantities can by divided into two categories -vectors and scalars.

A vector quantity is a quantity that is fully described by both magnitude and direction.

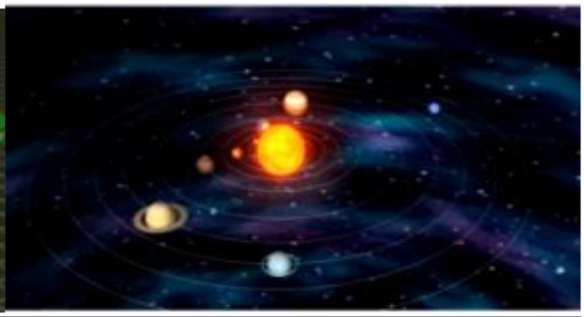
On the other hand, a scalar quantity is a quantity that is fully described by its magnitude.

The emphasis of this unit is to understand some fundamentals about vectors and to apply the fundamentals in order to understand motion and forces that occur in two dimensions.

In order to describe motion of an object in two dimensions (a plane) or three dimensions (space), we need to use vectors to describe the abovementioned physical quantities. Therefore, it is first necessary to learn the language of vectors.

What is a vector? How to add, subtract and multiply vectors?

What is the result of multiplying a vector by a real number?

We shall learn this to enable us to use vectors for defining velocity and acceleration in a plane.

SCALARS AND VECTORS

CLASSIFICATION OF PHYSICAL QUANTITIES:

Physical Quantities are classified into 'Scalars ' and 'Vectors'.

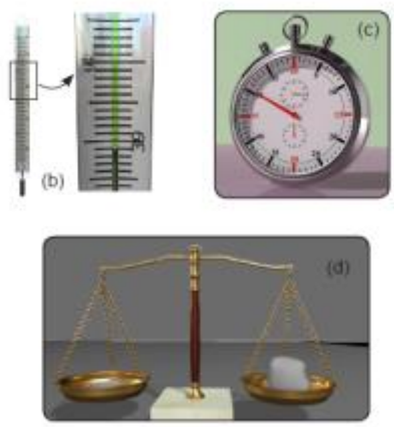
SCALAR QUANTITY:
A physical quantity having only magnitude but no direction is called a scalar.

For Example:

*1. Let us measure the **length of an object** AB by a scale.*
Here , the length of AB = 11.4 cm. It is a number.

*2. Let us measure the **temperature of water.***
*Here the temperature is equal to 30^0C**(degree centigrade)**.*
It is a number.

*3. Calculate **the time** (for example T= 50 min)*



Examples of scalars

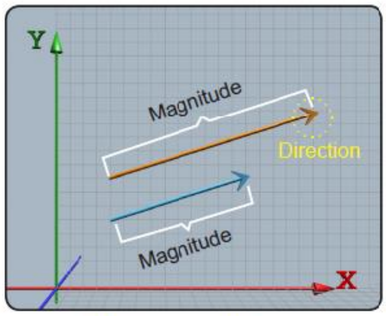
*4. Measure the **mass of an object** (for example m = 10 Kg).*

All these are numbers .
These physical quantities described completely by a number with proper units.These physical quantities are called 'scalars'. Length, time, mass, temperature, volume, density are some examples of scalars.

***VECTOR QUANTITY** :*

*Vector quantities have both **magnitude and direction.***

A physical quantity which has both magnitude and direction is called a **vector.**



Vector quantity

Example for scalar and vector quantities:
1.Distance and
2. Displacement
***Distance:** Distance is a scalar quantity, that refers the actual length of the path covered by a moving object.*
Here, we can see, an object is moving from one point to another along the path towards east . This path shows distance.

***Displacement:** Displacement is a vector quantity, it has both magnitude and direction. The magnitude of displacement is theshortest distance between the initial and final positions of the object. This vector shows the displacement of a moving object.*

<u>REPRESENTATION OF VECTORS:</u>

*Vector is geometrically represented by an **arrow**. Initial point of arrow is called **Tail** and the final point of arrow is called**head**.*

The length of the arrow is proportional to the magnitude of the vector.

Head of the vector gives the sense of direction.

***For example:** A displacement vector is represented by an arrow AB.*

The initial point (tail) of the vector is A. The final point (head) is B.

The length AB (measured to a scale) is the magnitude of the vector.

The direction of the vector is specified by the angle, the arrow makes with a reference line.

The magnitude of displacement AB is 30 m, the direction is 30^0(degrees) north of east.

*In written form, a vector is represented by a bold type letter'**d**' or $\vec{d}$*

*If magnitude of the vector is to be specified, one has to write either **|d|** or **d**.*

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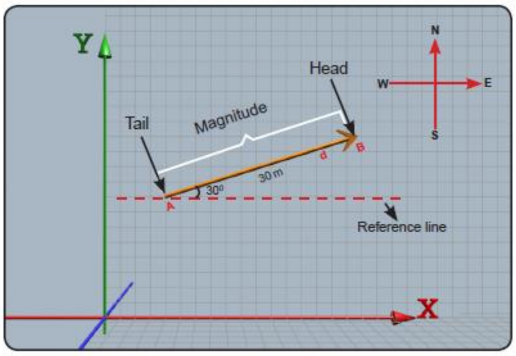
<u>TYPES OF VECTORS :</u>



Representation of vectors

<u>Equality of Vectors</u>

*In this module we will discuss about '**Equality of vectors'.***

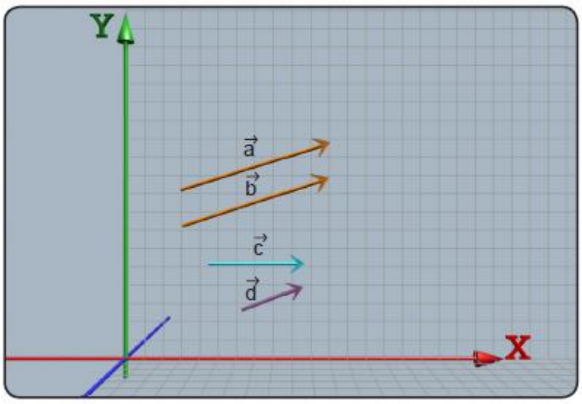
Two vectors are said to be equal only if they have same magnitude, same units and same direction representing same physical quantity.

*Let us consider some vectors **a, b, c** and **d**.*

*In these, **a** and**b** vectors have **samemagnitude** and indicates same **direction**.*

*Therefore, '**a**' and '**b**' are equal vectors .*

*And the vectors '**c**' and '**d**' have **different magnitude** and different direction. Therefore, these two vectors are not equal. Here, all of them have different initial points. We can move a vector parallel to itself without effecting it.*



Equality of vectors

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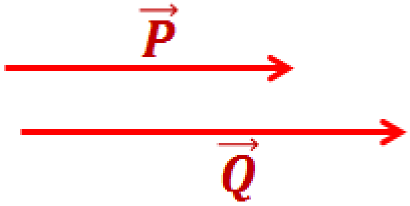
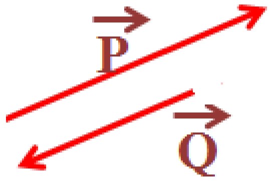
Parallel Vectors :

*Two or more vectors have same direction but their magnitudes may be different are called like **vectors.***



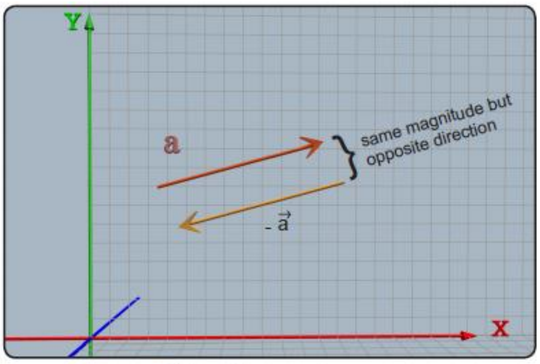
Anti-Parallel Vectors:

Two or more vectors act in opposite direction however there magnitudes may be different or called anti parallel vectors.



NEGATIVE VECTORS

*Negative of a vector is another vector having same magnitude but opposite direction.-**a**is the negative of vector '**a**' .These two vectors are mutually antiparallel.*



Negative vectors

POSITION VECTOR:

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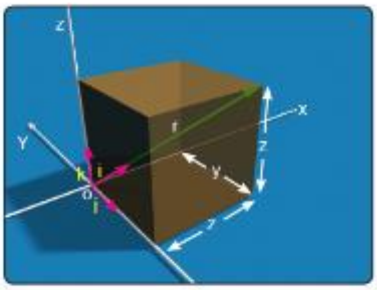
The position vector describes the position of a particle which is drawn from the origin of a reference frame, and it gives the complete information about the particle where it is located in the frame.

The position of a particle 'P' is indicated by a position vector OP with respect to the origin of *the co-ordinate system.*

The ordinates of particle 'P' are x, y and z or P(x,y,z).

The position vector OP is given by $OP = r = x\hat{i} + y\hat{j} + z\hat{k}$



Position Vector

Therefore, $|OP| = |r| = \sqrt{x^2 + y^2 + z^2}$

The unit vector along r is $r = \frac{r}{|r|} = \sqrt{\frac{xi + yj + zk}{x^2 + y^2 + z^2}}$

NULL VECTOR:

A vector of zero magnitude is called a null vector or zero vector. Its direction is indeterminate.

Null vector is represented by $\vec{O}$.

For example, The velocity of a particle at rest.Here, magnitude of velocity is zero. So, it is a null vector.

Another Example, Here, Car and ball are moving at constant velocities. We can observe, no acceleration in the car movement and ball movement. That is, In both cases magnitude of the acceleration is zero. Therefore, acceleration is a null vector.

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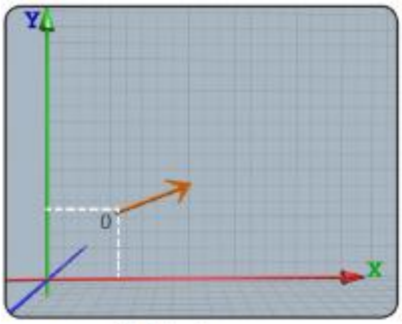

Null Vector

Examples of Null Vector

UNIT VECTOR:

A vector of unit magnitude is called unit vector. The unit vector in the direction of given vector is obtained by dividing the given vector with its magnitude.

A unit vector of $\vec{A}$ is written as A and is read as 'A cap' or 'A hat'. If $\vec{A}$ is a given vector, the unit vector in the direction of $\vec{A}$ is written as, $A = \frac{\vec{A}}{|\vec{A}|}$.

It is unitless and dimensionless vector and represent direction only.

In the right handed cartesian coordinate system. i,j and k are chosen as unit vectors along, the x-axis, y-axis and z-axis respectively. Thus i,j and k are called orthogonal unit vectors (orthonormal vectors).

ADDITION OF VECTORS:

Two vectors should not be added by normal methods. To add two vectors, we have to follow some rules.

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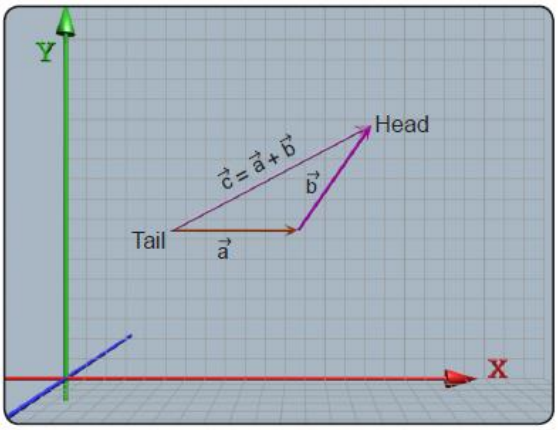
*Let us take two vectors **a**and**b**with their proper directions.Now, join the initial point (tail) of the second vector to the final point of the first vector.*

*Join the tail of **b** with the head of**a**.*

Then, join the initial point of first vector to the final point of second vector. This arrow represents sum of the two vectors (resultant vector).

*Now, Join the tail of a and the head of b gives the sum of **a**and **b**, that is **a + b***

*Therefore, sum of two vectors (resultant) **c = a + b***



Addition of vectors

LAWS OF VECTOR ADDITION:

commutativelaw of addition:

*Let us consider two vectors **a** and**b**. Now take equal vector of **b** and join the initial point of **a** and final point of **b**. That is sum of vector **a** and vector **b** (resultant is*

a** + **b**). Add two vectors **a** and**b** in different order, then obtain the same result. That is, **c = b + a

Therefore, the sum of the two vectors is independent of the order of the vectors. We can choose the two vectors in any order and get the same result.

a + b = b + a

*This is **commutative law of addition.***

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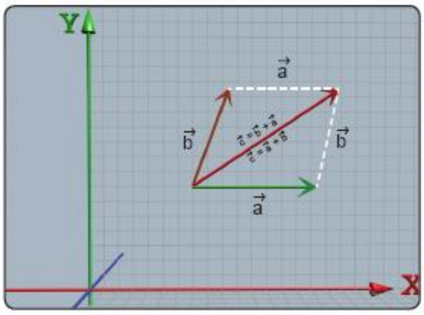


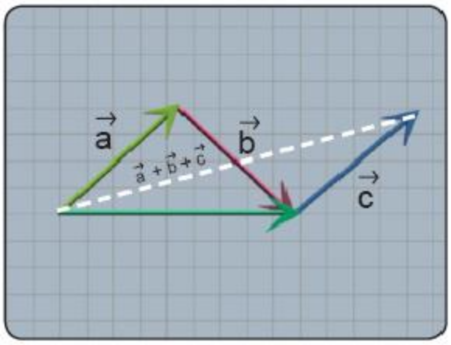
Commutative Law

ASSOCIATIVE LAW OF ADDITION:

*The resultant of **a** and **b** is **a + b** and add that third vector '**c**' to the resultant **a + b** . Then the resultant (sum) of **(a + b)** and **c** is represented by resultant vector **(a + b) +c**. The sum of '**b**' and '**c**' represented by resultant **(b + c)** and add third vector **a** to**(b+ c)**. Then obtain the same result. that is **a + (b + c)***

(a + b) + c = a + (b + c)

Law of vector addition obeys Associative law.



Associaive Law

<u>RESOLUTION OF VECTORS</u>

<u>Resolution of a vector into components:</u>

Let us consider two vectors 'a' and 'b' together (have same initial points). Then vector 'R' is resultant to vectors 'a' and 'b'. The single vector 'R' is resolved into two other vectors. The given vector 'R' is said to be resolved into two components, say along X-axis and the other along Y-axis. The vector 'R' is represented by AD. Make an angle 'θ' with the X-axis. Draw a perpendicular line DB to the X-axis and draw a perpendicular line DC to the Y-axis

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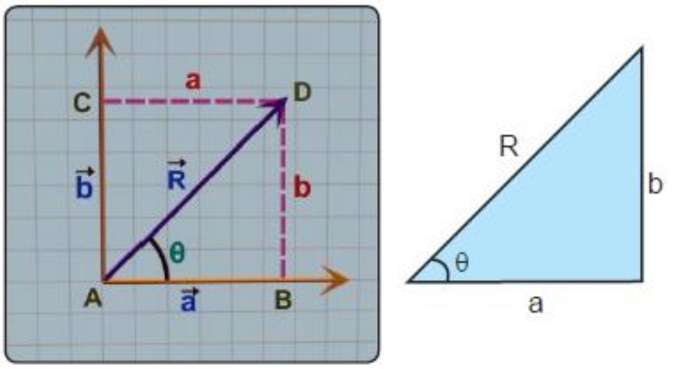


Resolution of vectors

X-component of R is 'a'.

Y-component of R is 'b'.

Now, $\cos\theta = \dfrac{a}{R}$

And $R\cos\theta = a$

Or $R_x = R\cos\theta$

And $\sin\theta = \dfrac{b}{R}$

Therefore $R\sin\theta = b$

Or $R_y = R\sin\theta$

Rectangle Components

Rectangle components of a vector,

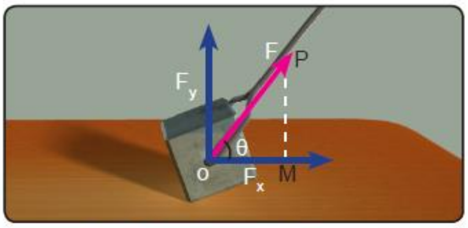
*Here it is a vector '**F**'.*

Now, construct a triangle 'OPM'

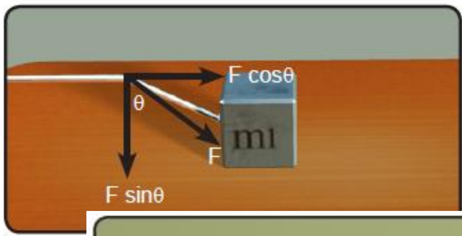
in the triangle OPM

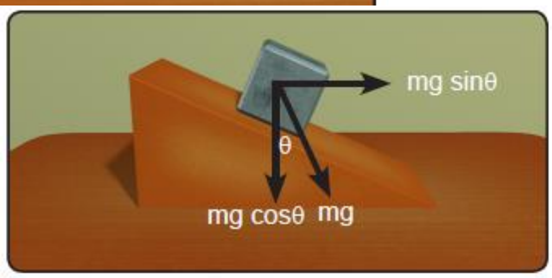
$\cos\theta = \frac{F_x}{F}$ *(cos theta is equal to* F_x *divided by F)*

$F_x = F\cos\theta$

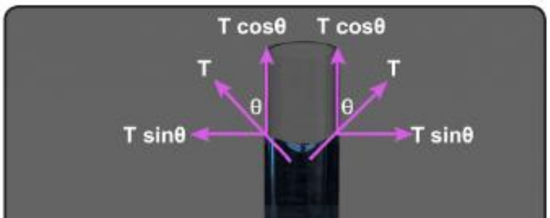


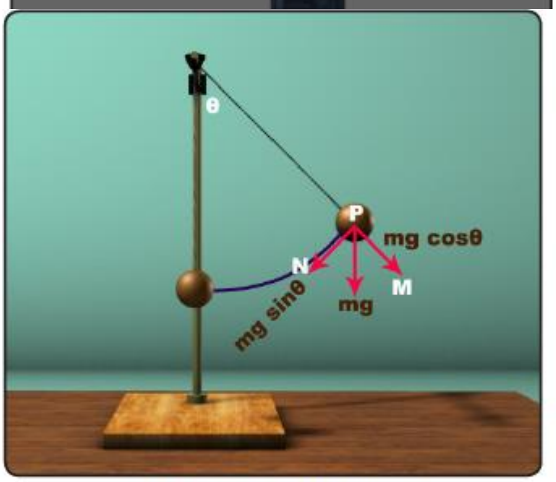
Ex: Pushing an object





Ex: Inclined Plane





Ex: Surface tension

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Horizontal component (F_x) of vector 'F' isF_x= F cosθ

On the triangle 'OPM'

$sin\ \theta = \frac{F_y}{F}$

$$F_y = Fsin\theta$$

Vertical component of vector 'F' is $F_y = F\ sin\ \theta$

Examples of Rectangle components..

1) In pulling an object, the ***force vector*** *is resolving into rectangular components,*

Horizontal component $F_x = F\ cos\ \theta$

Vertical Component $F_y = F\ sin\ \theta$

2) In pushing an object, the force vector is resolving into rectangular components,

Horizontal component $F_x = F\ cos\ \theta$

Vertical Component $F_y = F\ sin\ \theta$

3) In motion of a body on an inclined plane, weight of the body is divided into rectangular components. They are

mgcos θ is perpendicular to the plane and

mg sin θ is down the plane.

4) In determination of surface tension by capillary rise method, the force 'T' is resolving into two rectangular components as shown in the figure.

Horizontal component isTsin θ

Vertical componentisTcos θ

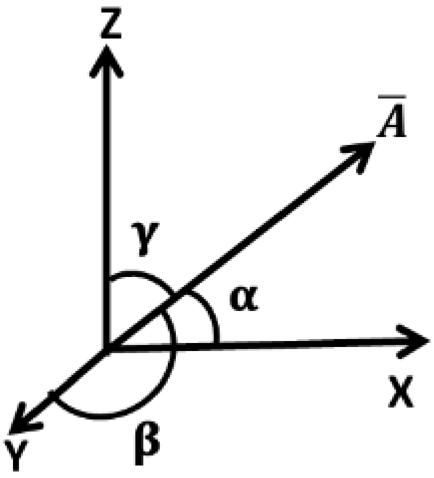
5) In simple pendulum experiment,

resolving the weight (mg) of bob, into two rectangular components.

mgcos θ along $\overline{PM}$,

mg sin θ along $\overline{PN}$ (since $\overline{PN}$ is perpendicular to $\overline{PM}$).

DIRECTION OF COSINES:



- *If α, β and γ are the angle made by $\vec{A}$ with X- axis, Y- axis and Z-axis respectively, then*

  $cos\alpha = \frac{A_x}{A}; cos\beta = \frac{A_y}{A}; cos\gamma = \frac{A_z}{A}$

  $and cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

  $sin^2\alpha + sin^2\beta + sin^2\gamma = 1$

- *If cosα=l ,cos β=m and cosγ=n, then l, m, n are called direction cosines of the vector.*

  $l^2 + m^2 + n^2 = 1$

- *If vectors $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$ are parallel, then $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$ and $\vec{A} = K\vec{B}$ where K is a scalar.*

  - ***Lami's theorem:****When three coplanar forces $\vec{P}, \vec{Q}$ and $\vec{R}$ keep a body in equilibrium, then*

    $$\frac{\boldsymbol{P}}{\boldsymbol{sin\alpha}} = \frac{\boldsymbol{Q}}{\boldsymbol{sin\beta}} = \frac{\boldsymbol{R}}{\boldsymbol{sin\gamma}}$$

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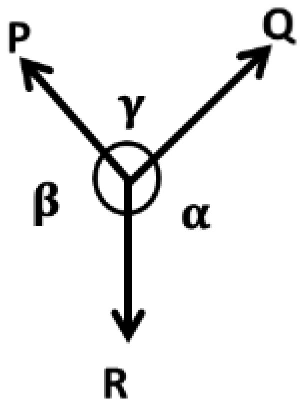


- When a number of force acting on a body keep in equilibrium, then the algebraic sum of the components along the X- components along Y- direction is also equal to zero. i.e., $\sum F_x$=0 and $\sum F_y$=0.
- If N forces $\overline{A_1}, \overline{A_2}, \overline{A_3} + \cdots \overline{A_N}$ are acting on a point such that $\overline{A_1}, \overline{A_2}, \overline{A_3} + \cdots \overline{A_N}$ =0 and $A_1 = A_2 = A_3 = \cdots A_N$, then the adjacent vectors are inclined to each other at an angle $\frac{2\pi}{N}$ or $\frac{360^0}{N}$.
- N forces each of magnitude F are acting ona point and angle between any two adjacent forces is θ, then resultant force $F_{resultant} = \frac{F\, sin\left(\frac{N\theta}{2}\right)}{sin\left(\frac{\theta}{2}\right)}$.

SUBTRACTION OF VECTORS:

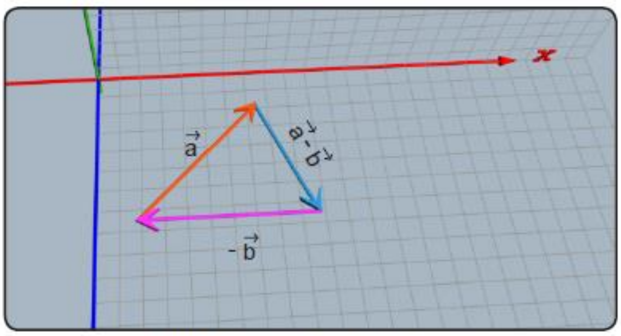
Let us consider two vectors, $\bar{a}$ and $\bar{b}$.

When we subtract two vectors, simply reverse the vector to be subtracted, and then add them.

*Here, If vector '**b**' is to be subtracted from vector '**a**'*

*Then, the negative vector of '**b**'is added to vector '**a**'.(that is -$\bar{b}$ is added to $\bar{a}$)*

It is represented like this, $\bar{a}$ +(-$\bar{b}$) = $\bar{a}$ - $\bar{b}$



Subtraction of vectors

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PROPERTIES:

- *Commutative law does not hold good in vector subtraction.*
  $\vec{a}-\vec{b}\neq\vec{b}-\vec{a}$
- *Associative law does not hold good in vector subtraction.* $\left(\vec{a}-\vec{b}\right)-\vec{c}\neq\vec{a}-\left(\vec{b}-\vec{c}\right)$

****

Triangle law of vectors:

*In this topic we will discuss about '**Triangle law of vectors'**.*

Statement:
If two vectors are represented by the sides of a triangle both in magnitude and direction taken in order, the resultant (sum) of the vectors is given by the closing (third) side of the triangle taken in the reverse order both in the magnitude and direction.

Let an object moving from point A to point B and from point B to point C.

The displacement of the object from A to B is represented by the vector $\overrightarrow{AB}$ *or* $\vec{a}$.

The displacement of the object from B to C is represented by the vector $\overrightarrow{BC}$ *or* $\vec{b}$.

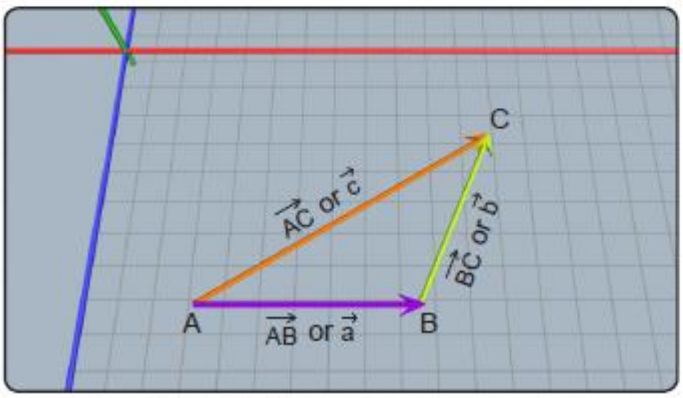
Now, join the initial point (A) of the first vector and final point (C) of second vector to get the sum or resultant vector of two displacement vectors.

Vector $\overrightarrow{AC}$ *is the sum or resultant vector of the displacement vectors* $\overrightarrow{AB}$*and* $\overrightarrow{BC}$.

Therefore, $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

*This is known as **Triangle law of vectors** .*

This triangle law of vectors is used for addition of two vectors.



Triangle law of vectors

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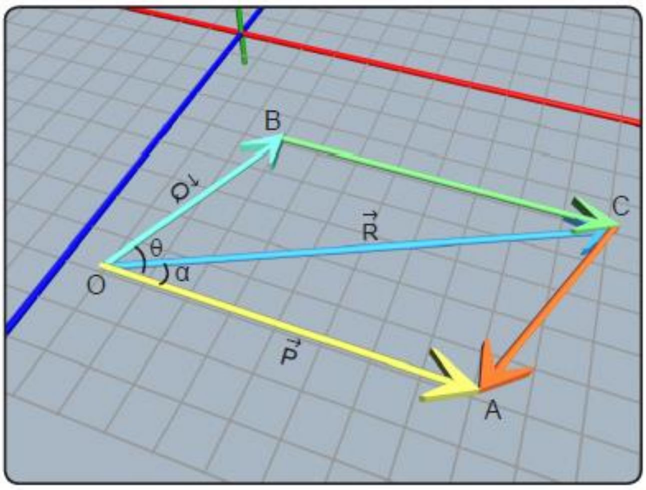
Paralleogram law of vectors

Statement: *If two vectors are represented in magnitude and direction by the two adjacent sides of a parallelogram, drawn from a point and their resultant is represented in magnitude and direction by the diagonal passing through the same point.*

Let $\vec{P}$,$\vec{Q}$ *be two vectors and they have common initial point 'O'.Let '*θ*' be the angle between two forces (two vectors).*

OA represents the vector 'P' and OB represents the vector 'Q'.
A parallelogram OACB is constructed using these vectors as adjacent sides.
A diagonal OC is drawn from the point 'O'.
Diagonal OC represents the resultant 'R'.
That is, $\vec{R} = (\vec{P} + \vec{Q})$
Angle made by OC with OA, is 'α*'*
Therefore, Direction of resultant is 'α*'.*
Magnitude of R = OC.
Let us extend the line OA upto point 'O' and draw a perpendicular from 'C' on to 'D'.
From the parallelogram OBCA,
$\angle BOA = \angle CAD = \theta$ *OA = BC = P (The magnitude of P)*
OB = AC = Q (The magnitude of Q)
and OC = R (the magnitude of resultant R)
In right angle triangle ODC.



Parallelogram law

$OC^2 = OD^2+DC^2$*(By Pythagoreans theorem)*
$R^2 = (OA + AD)^2 + DC^2$
$R^2= (OA)^2+ (AD)^2+ 2OA.AD+DC^2$ ----------------- → *Equation (1)*
Component of AD = AC $\cos\theta$ *= Q*$\cos\theta$ --------------→ *Equation (2)*
Component of DC = AC $\sin\theta$ *= Q* $\sin\theta$ -------------→ *Equation (3)*
Hence, $AD^2+ DC^2 = Q^2 sin^2\theta + Q^2 cos^2\theta$

$$AD^2 + DC^2 = Q^2$$ ---------------->*Equation (4)*

Substituting equations 2 & 4in 1

$R^2 = OA^2 + 2\, OA.AD + Q^2$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\therefore R = \sqrt{P^2 + Q^2 + 2PQcos\theta}$

Therefore, The magnitude of the resultant R depends not only on the magnitude of the vector P and Q but also the angle between them.

The resultant 'R' makes an angle 'α' with OA vector P.

'α' indicates the direction of 'R'.

From Triangle OCD,

$Tan\alpha = \frac{DC}{OD} = \frac{DC}{OA+AD}$ *(From equations 2 & 3)*

$\therefore Tan\alpha = \frac{QSin\theta}{P+Qcos\theta}$

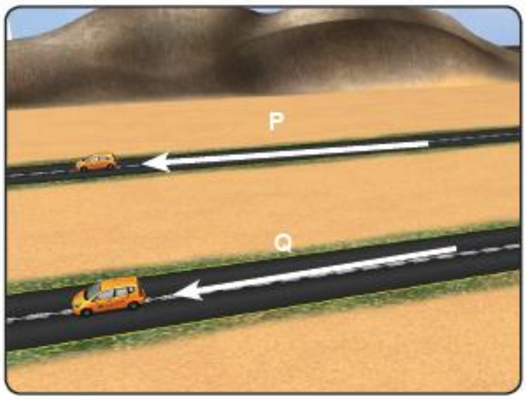
$\alpha = Tan^{-1}\left(\frac{QSin\theta}{P+Qcos\theta}\right)$

∴ Magnitude of the resultant

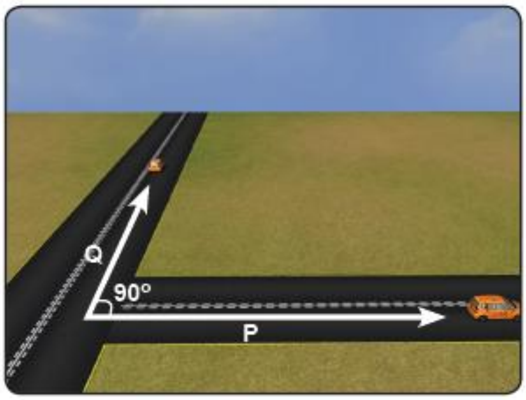
$$R = \sqrt{P^2 + Q^2 + 2PQcos\theta}$$

Direction of the resultant

$\alpha = Tan^{-1}\left(\frac{QSin\theta}{P+Qcos\theta}\right)$



Case 1 : When P and Q are parallel



Case 2 : When P and Q are perpendicular

Case 1: *If P and Q are parallel, that is angle between them is zero. Then*

$R = \sqrt{P^2 + Q^2 + 2PQcos\theta}$

$= \sqrt{P^2 + Q^2 + 2PQcos\,0}$ *(since* $\cos\theta = \cos 0 = 1$*)*

therefore $R = \sqrt{(P+Q)^2} = P + Q$ *The magnitude of the resultant is the sum of the magnitude of the two vectors and the resultant is directed in the same direction as the two vectors.*

Case 2: *If P and Q are mutually perpendicular, that is* $\theta = 90^0$*(degrees).*

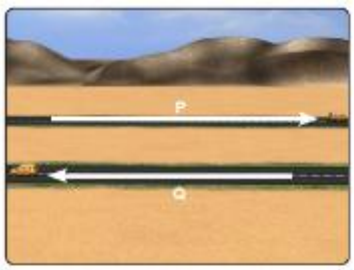
$R = \sqrt{P^2 + Q^2 + 2PQcos\theta}$ *, here* $\cos 90^0 = 0$*, then this equation becomes*

$R = \sqrt{P^2 + Q^2}$ *And* $Tan\,\alpha = \frac{Q}{P}$ *, that is* $\alpha = Tan^{-1}\left(\frac{Q}{P}\right)$

Case 3: *If P and Q are anti-parallel, that is* $\theta = 180^0$*, and* $Cos180^0 = -1$

Then $R = \sqrt{P^2 + Q^2 + 2PQcos\theta}$

$= \sqrt{P^2 + Q^2 - 2PQ} = \sqrt{(P-Q)^2} = P - Q$



Case 3 : When P and Q are anti-parallel

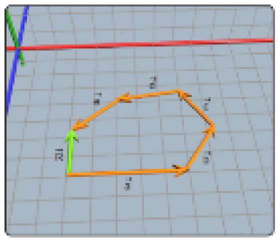
MOTION IN APLANE

$R = P - Q$ if $P > Q$ and $\alpha = 0^0$

$R = Q - P$ if $Q > P$ and $\alpha = 180^0$

When vectors are anti-parallel the magnitude of the resultant is equal to the difference of the magnitudes of the two vectors and its direction is along the vector whose magnitude is greater.

POLYGON LAW OF VECTORS :



Polygon law of vectors

*In this module we will discuss about '**Polygon law of vectors**.*

Statement:*If a number of vectors are represented as the sides of a polygon both in magnitude and direction taken in order, the resultant is represented by the closing side of the polygon taken in the reverse order both in magnitude and direction.*

Polygon law of vectors is used for addition of more than two vectors.

Let us consider five vectors $\vec{a}$, $\vec{b}$, $\vec{c}$,$\vec{d}$, *and* $\vec{e}$.

Now, displace the vector $\vec{b}$ *parallel to itself till the tail of '**b**' touches the head of '*$\vec{a}$'.

In the same way, move the vector ($\vec{c}$) *parallel to itself till the tail of vector **'c'** touches the head of vector **'b'**.*

*Again, displace the vector **d**, parallel to itself till the tail of vector**'d'** touches the head of vector **'c'.***

*And displace the vector **'e'** parallel to itself till the tail of vector **'e'** touches the head of vector **'d'.***

*Now, a vector drawn from tail of vector '**a**' to the head of vector '**e**'. This will be the sum or resultant of vectors* $\vec{a}$, $\vec{b}$, $\vec{c}$,$\vec{d}$, *and* $\vec{e}$.

$Therefore, \quad R = \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e}$

This is polygon law of vectors.

Multiplication of Vectors:

In this module we will discuss about 'Multiplication of vectors'.

Two vectors a and b can be multiplied in two ways.

MOTION IN APLANE

One way of multiplication produces a scalar as a product and the other way gives a vector as product.

Multiplication of two vectors which produces a scalar is called the scalar product or dot product.

$\vec{a} . \vec{b}$ *= scalar.*

The second type of multiplication which produces a third vector is called vector product or cross product.

Vector 1 × Vector 2 = Vector 3

Types of vector multiplication:

1.Scalar Product.

2.Vector Product.

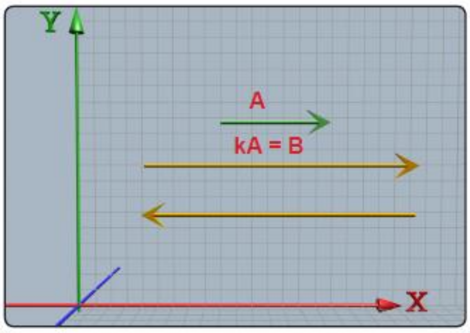
Multiplication of a vector by a scalar:

A vector can be multiplied by a scalar.

If a vector 'A' is multiplied by a scalar k, then obtained a new vector 'B'.

It is a new vector B, equals to kA .

It has the same direction as A and magnitude is 'k' times the magnitude of A .



Multiplication of vector by a scalar

If a vector A is multiplied by a scalar '4', then the new vector is 4A.

It has same direction as 'A'.

Magnitude of B is 4 times the magnitude of A .

If scalar 'k' is negative, the vector 'B' has the opposite direction of 'A'.

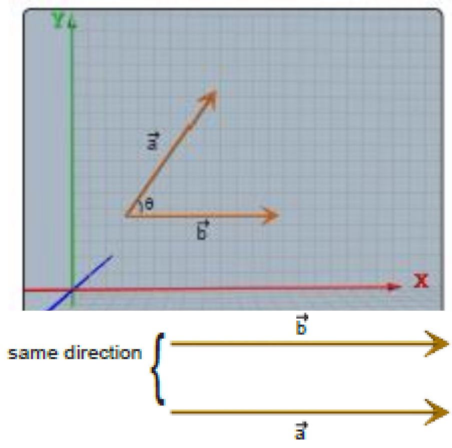
If Scalar k = 0 then the new vector B is a null vector.

Scalar Product or Dot Product

Let us take two vectors a and b

Definition:\
Scalar product of two vectors is

defined as the product of the magnitude of a



Vectors $\vec{a}$ and $\vec{b}$ same direction *and magnitude of b, and cosine angle between*

a and b.

Therefore, $a \cdot b = |a|\,|b|\,\cos\theta$

$= ab\cos\theta$

If vectors a and b are in the same direction :

That is, $\theta = 0$ *degrees*

$a.b = ab\cos 0$ *(a dot b equal to abcos zero)*

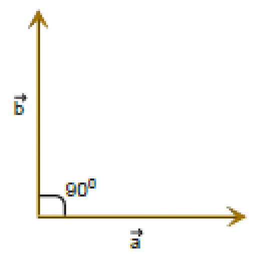
$= ab$ ***(sincecos zero = one)***

If Vectors a and b are perpendicular to each other:

That is, $\theta = 90^0$

Therefore, $a.b = ab\cos 90^0 = 0$ ***(since cos 90 = zero)***

Then, a and b are said to be orthogonal.



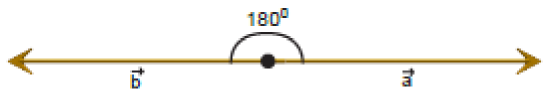
Vectors $\vec{a}$ and $\vec{b}$ perpendicular to each other

If a, b are anti parallel then the scalar product can be negative:

That is, $\theta = 180^0$

$a.b = a\,b\cos 180^0$

$= -a\,b$ *(since* $\cos 180^0 = -1$*)*



Vectors $\vec{a}$ and $\vec{b}$ anti parallel

Example:

Scalar product between force and displacement produces work

$W = F.S$ *(W is equal to F dot S)or*

The dot product of force and displacement gives work. This is a scalar quantity.

Scalar product between force and velocity produces power,

$P = F.V$ *or*

The dot product of force and velocity gives the power. Power is a scalar quantity.

Vector product (or) cross product:

If two vectors are multiplied such that their product is again a vector, the product is called vector product or cross product.

MOTION IN APLANE

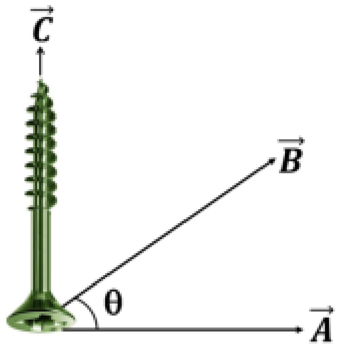
Definition: *The vector product of two vectors is defined as a vector having magnitude equal to the product of their magnitudes and the sine of the angle between them and direction perpendicular to the plane containing the two vectors in accordance with right handed system*

If $\vec{A}$ and $\vec{B}$ *are two vectors inclined at an angle* $'\theta'$ *mutually, their vector product is expressed as* $\vec{A}\times\vec{B}=\vec{C}$

The magnitude of $\vec{C}$ *is* $|\vec{C}| = |\vec{A}\times\vec{B}| = |\vec{A}|\,|\vec{B}|\sin\theta$

The direction of $\vec{C}$ *is perpendicular to the plane containing both* $\vec{A}$ and $\vec{B}$, *obeying right handed screw rule or right hand thumb rule (right handed system) which are explained below.*

i) *Right hand screw rule: A right hand screw is set with its axis perpendicular to the plane containing vectors* $\vec{A}$ and $\vec{B}$, *If the screw is rotated in a direction from* $\vec{A}$ to $\vec{B}$, *through the smaller angle between them, the direction in which the axis of screw advances, gives the direction of* $\vec{C}$ *(fig) it may be noted*



Properties of vector product

1. *The cross product does not obey commutative law. The order of the vector in the product is of great importance, change of order alters the sign of the cross product. i.e* $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ *but* $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$

2. *The cross product obeys distributive law i.e* $\vec{A}\times(\vec{B}\times\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

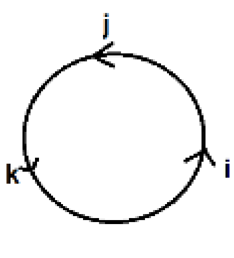
3. *If* $\vec{A}$ and $\vec{B}$ *are collinear (i.e* θ*=0*^{0} *and 180*^{0}*) then their vector product yields null vectors* $\vec{A}\times\vec{B}=\vec{O}\,[\sin\theta=0]$

4. *If* $\vec{A}$ and $\vec{B}$ *are mutually perpendicular, their cross product is a vector of maximum magnitude .it is the product of their magnitudes* $|\vec{A}\times\vec{B}| = AB\sin 90^0 = AB$

5. *The cross product of a vector with itself yields null vector i.e* $\vec{A}\times\vec{A}=\vec{O}$

6. *If* $\vec{i},\vec{j}$ and $\vec{k}$ *are the unit vectors along the axes of right handed Cartesian coordinate system.*

   $i\times i=j\times j=k\times k=0$



MOTION IN APLANE

$i \times j = k \Rightarrow j \times i = -k$

$j \times k = i \Rightarrow k \times j = -i$

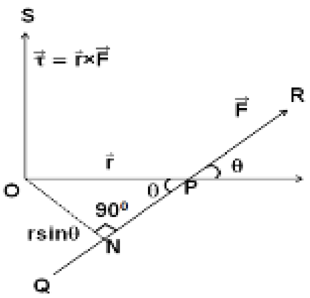
$k \times i = j \Rightarrow i \times k = -j$

Note: the characteristics 1,2 and 3 above may be termed as "laws of vector product".

Examples of cross product

Some physical quantities which can be expressed as cross product are mentioned below.

Torque (or) moment of force: It is the cross product of position vector and force vector. ie $\vec{\tau} = \vec{r} \times \vec{F}$



If fig, the directed line segment QR represents a force vector in magnitude and direction. 'P' → is a point on the line of action F and r is the position vector of 'P' w.r.t an arbitrary point 'O'. V is the perpendicular drawn from 'O' on to QR. The angle between $\vec{r}$ *and F is* θ

From triangle OPN; one can write, $\frac{ON}{OP} = \sin\theta \Rightarrow ON = OP\sin\theta = On = |r|\sin\theta$

The moment of force about the point 'O' is the product of the force and the normal distance of 'O' from the line of action of force.

Thus the magnitude of moment of force = (ON)F $= (ON)F \Rightarrow \tau = (r\sin\theta)F$

In Vector from $\vec{\tau} = \vec{r} \times \vec{F}$

The direction of $\vec{\tau}$ *is along the upward normal (OS) to the plane containing,* $\vec{r}$ *and* $\vec{F}$ *obeying the thumb rule.*

2) Linear velocity of rotating body: *It is the cross product of angular velocity and position vector, i.e* $\vec{V} = \vec{\omega} \times \vec{r}$

RELATIVEVELOCITY:

The velocity of one body with respect to another body is called relative velocity. Let us consider two bodies A and B. $\vec{V}_A$ *and* $\vec{V}_B$ *are the velocities of two bodies A and B. If two bodies are moving in the same direction. The relative velocity of A with respect to B is denoted by* $\vec{V}_{AB}$*, and the relative velocity of B with respect to A is denoted by* $\vec{V}_{BA}$.

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

Relative velocity in parallel motion

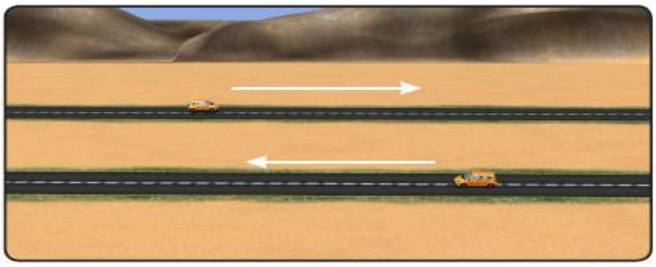
$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

If two bodies are moving in opposite direction then the velocity of the one body is negative. Then the relative velocity of A with respect B is

$\vec{V}_{AB} = \vec{V}_A - (-\vec{V}_B) = \vec{V}_A + \vec{V}_B$

and the velocity of B with respect to A is

$\vec{V}_{BA} = \vec{V}_B - (-\vec{V}_A) = \vec{V}_B + \vec{V}_A$



Relative velocity in Anti- parallel motion

Cases:

(i) Parallel motion, $\theta = 0^0$

Relative velocity $V_{relative} = \vec{V}_A - \vec{V}_B$ or

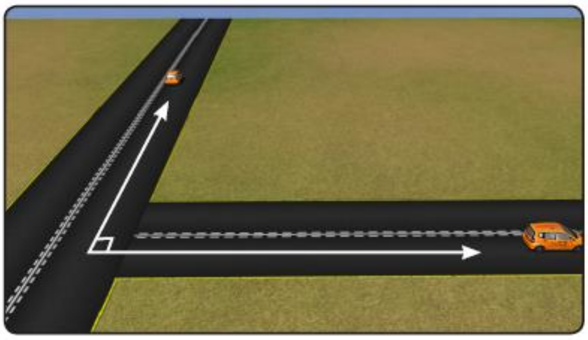
$\vec{V}_{rel} = \vec{V}_B - \vec{V}_A$

(ii) Anti-parallel motion, $\theta = 180^0$.

Relative velocity $\vec{V}_{rel} = \vec{V}_A + \vec{V}_B$

(iii)Motion at right angle to each other, $\theta = 90^0$,

Relative velocity $\vec{V}_{rel} = \sqrt{\vec{V}_A^{\,2} + \vec{V}_B^{\,2}}$

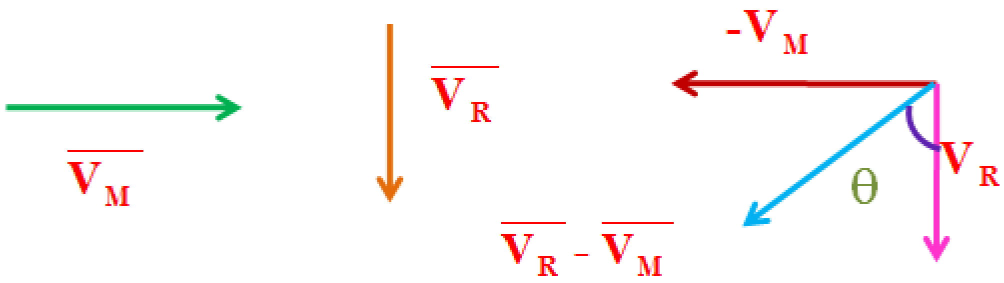


Relative velocity in right angle

Relative velocity of rain:

- *Rain is falling vertically downwards with velocity $\overline{V_R}$.*
- *A man is walking horizontally on a road with velocity $\overline{V_M}$*
- *In order to protect himself from rain he must hold his umbrella in the direction*

at relative velocity of rain w.r.t him.



1) Magnitude of relative velocity of rain w.r.t man is $|\overline{V_R} - \overline{V_M}| = \sqrt{V_R^2 + V_M^2}$

2) Angle made by an umbrella to the vertical is θ = $\tan^{-1}\left(\frac{V_M}{V_r}\right)$

Relative velocity of swimmer :

If a man can swim relative to water velocity $\bar{V}$ *and water is flowing relative to ground with velocity* $\overline{V_M}$*, velocity of man relative to ground* $\overline{V_R}$ *will be given by*

$\bar{V} = \overline{V_M} - \overline{V_R}$ *i.e* $\overline{V_M} = \bar{V} + \overline{V_R}$

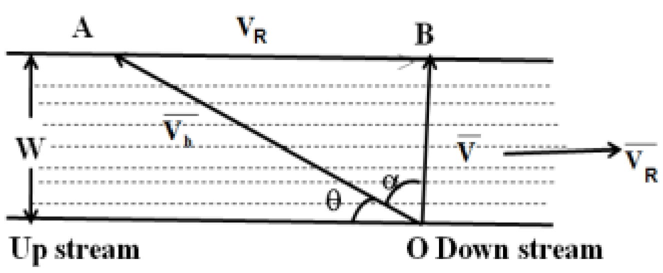
a)If the swimming is in the direction of flow of water , then $V_M = V + V_R$

b)If the swimming is opposite to the flow of water , then $V_M = V - V_R$

Motion of a boat across a river:

Suppose the river is flowing with velocity $\overline{V_R}$ *and the velocity of the boat in still water is* $\overline{V_b}$ *and 'W' be the width of the river.*

He is standing on one bank of the river and wants to cross the river then two cases arise.



Here OAB is the triangle of vectors

$\overline{OA} = \overline{V_b}$, $\overline{AB} = \overline{V_R}$ ***their resultant is given by*** $\overline{OB} = \overline{V}$

From the triangle OBA we find

$\cos\theta = \frac{V_R}{V_b}$ ***also*** $\sin\alpha = \frac{V_R}{V_b}$

Time taken to cross the river will be given by

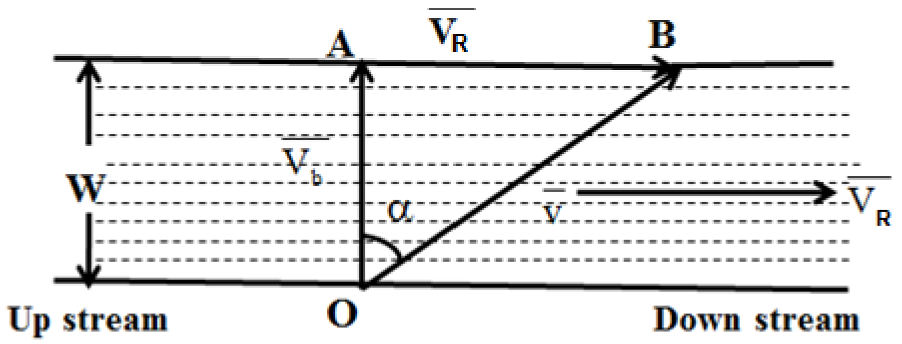
MOTION IN APLANE

$$t = \frac{W}{V} = \frac{W}{\sqrt{V_b{}^2 - V_R{}^2}}$$

Case(i):To cross the river along shortest path:

He has to row the boat along $\overline{OA}$ making angle αWith the upstream as shown

Case(ii):To cross the river in shortest possible time:



The boat should be rowed perpendicular to the bank.

- *The time taken to cross the river will be given by*

$$t = \frac{W}{V_b}$$

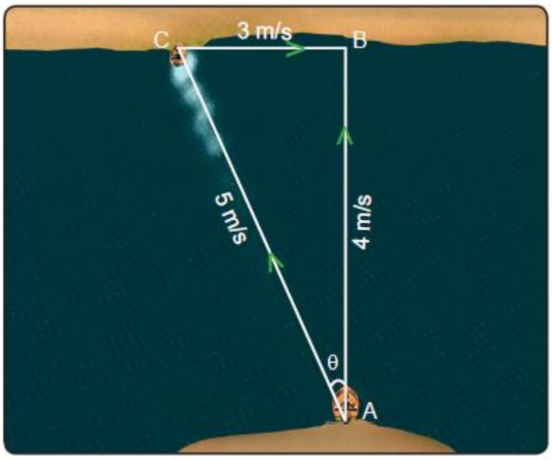
- *In this case the boat reaches the opposite bank at a distance AB down stream. This distance AB is called shift or drift of boatdown stream.*

$$\text{Drift } AB = W\left[\frac{V_R}{V_b}\right]$$

***Example:**A boat with a speed of 5 m/s crosses a river flowing at 3 m/s to a spot directly opposite on the other bank, 400 m (meters) away. What is the direction in which it must head? What is the time it takes to cross the river?*
Let the boat head at an angle 'θ'
The resultant velocity is $R=\sqrt{u^2+v^2}$ *= 4 m/s*
The boat must head at an angle, sin θ = 3/5 ,

$$\theta = \sin^{-1}\frac{3}{5} = 37^0$$

Therefore the boat must head in 37^0 in the backward direction andTime taken to cross the boat is (t) = distance/ resultant velocity = 400/4 = 100 seconds



Example of Traingle law

MOTION IN A PLANE:

In this section we shall see how to describe motion in two dimensions using vectors.

POSITION VECTOR

The position vector r of a particle P located in a plane with reference to the origin of an x - y reference frame is given by

$$\vec{r} = x\hat{i} + y\hat{j}$$

where x and y are components of $\vec{r}$ along x- axis, and y- axes or simply they are the coordinates of the object

DISPLACEMENT

Suppose the particle is at point P at time t and P| at time t|. The displacement is

$$\Delta\vec{r} = \overrightarrow{r^1} - \vec{r}$$

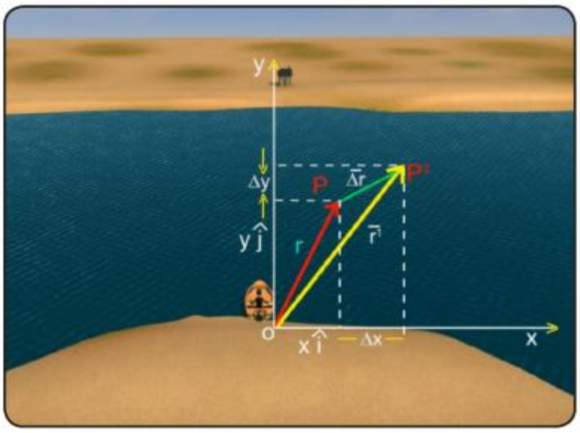
In component form,

$$\Delta\vec{r} = (x^1\hat{i} + y^1\hat{j}) - (x\hat{i} + y\hat{j})$$

$$\Rightarrow \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$$

Where, $\Delta x = x^1 - x$

$\Delta y = y^1 - y$



Motion in a plane

VELOCITY

The average velocity ($\bar{v}$) of an object is the ratio of the displacement and the corresponding time interval:

MOTION IN APLANE

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$

$$= \frac{\Delta\vec{x}}{\Delta t}\hat{i} + \frac{\Delta\vec{y}}{\Delta t}\hat{j}$$

$$= \Delta\vec{v} = \Delta\overrightarrow{v_x}\hat{i} + \Delta\overrightarrow{v_y}\hat{j}$$

The instantaneous velocity is given by the limiting value of the average velocity as the time interval approaches zero i.e.,

$$\vec{v} = \lim_{\Delta t\to 0} = \frac{\Delta\vec{r}}{\Delta t}\frac{d\vec{r}}{dt}$$

ACCELERATION

The average acceleration a of an object for a time interval Δt moving in x-y plane is the change in velocity divided by the time interval:

$$\vec{v} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta\overrightarrow{v_x}\hat{i} + \Delta\overrightarrow{v_y}\hat{j}}{\Delta t}$$

$$= \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j}$$

The instantaneous acceleration is the limiting value of the average acceleration as the time interval approaches zero i.e.,

$$\vec{a} = \lim_{\Delta t\to 0} = \frac{\Delta\vec{v}}{\Delta t}\frac{d\vec{v}}{dt}$$

$$\vec{a} = \lim_{\Delta t\to 0} = \frac{\Delta\overrightarrow{v_x}\hat{i} + \Delta\overrightarrow{v_y}\hat{j}}{\Delta t}$$

$$\vec{a} = \hat{i}\lim_{\Delta t\to 0} = \frac{\Delta v_x}{\Delta t} + \hat{j}$$

$\vec{a} = \lim_{\Delta t\to 0} = \dfrac{\Delta v_y}{\Delta t} \Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j}$ *where,* $a_x = \dfrac{dv_x}{dt}$ $a_y = \dfrac{dv_y}{dt}$

MOTION IN APLANE

MOTION IN A PLANE WITH CONSTANTACCELERATION

Motion in a plane with constant acceleration and relative velocity in two Dimensions:

*Suppose that an object is moving in x−y plane and its acceleration **a** is constant. Over an interval of time, the average acceleration will equal this constant value.*

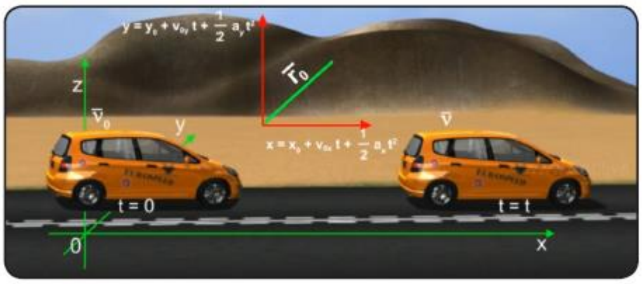
Let , $\vec{v_0} \rightarrow$ *Initial velocity of the object at time t = 0*

$\vec{v} \rightarrow$ *Final velocity of the object at time t = t*

Then,

$$\vec{a} = \frac{\vec{v} - \vec{v_0}}{t - 0} = \frac{\vec{v} - \vec{v_0}}{t}$$

$$\Rightarrow \vec{v} = \vec{v_0} + \vec{a}t \rightarrow Eq(1)$$



Relative Velocity in Two Dimensions

In terms of components,

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

Let $\vec{r_0}$ *be the position vector of the particle at time 0 and r be the position vector of the particle at time t.*

The displacement is the average velocity multiplied by the time interval.

$$\vec{r} - \vec{r_0} = \frac{\vec{v} + \vec{v_0}}{2} t \frac{\vec{v_0} + \vec{a}t + \vec{v_0}}{2} \times t \rightarrow Eq(1) \quad \vec{r} - \vec{r_0} = \vec{v_0}t + \frac{1}{2}\vec{a}t^2 \quad \vec{r} = \vec{r_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2 \rightarrow Eq(2)$$

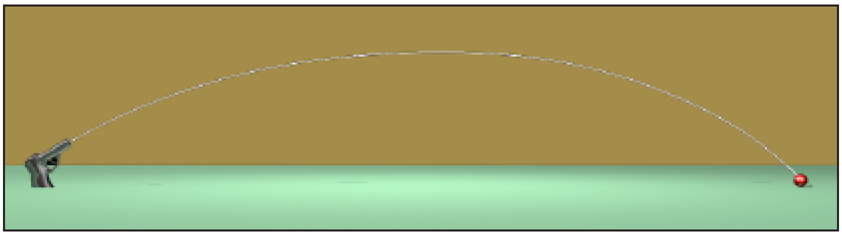
In component form:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \rightarrow Eq(3) \qquad y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \rightarrow Eq(4)$$

Motion in a plane can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.

MOTION IN APLANE

PROJECTILE



Ex: of Projectile

A projectile is a body projected with an initial velocity directed at an angle other than 90^0(degrees) with the horizontal and moves under only the force** "gravity" **and no other forces acts on it.

A body thrown with an angle with the horizontal is called a ***"Projectile".***

The path traced by a projectile is called ***"trajectory"*** and is a parabola.

***A body projected at** 'o' **with a initial velocity** 'u' **and it makes an angle with it. Initial velocity of the object is** 'u'.*

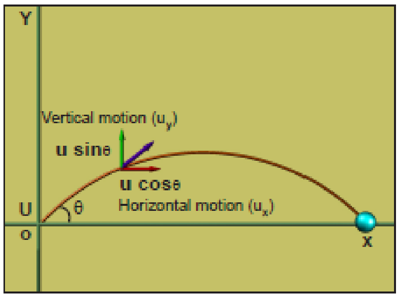
The projectile motion can be resolved into two separate straight line motions. The components are:

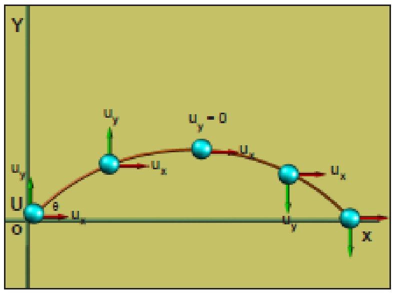
1. Horizontal motion with zero acceleration.

2.Vertical motion with constant downward acceleration.

Horizontal component is parallel to X–axis whereas vertical component is perpendicular to X–axis (Parallel to Y-axis).

Object projected with an initial velocity** 'u' **and that makes an angle θ with X-axis.





Ex: Projectile

The velocity of projection** u, **can be resolved into horizontal and vertical components** u_x **and** u_y **respectively and are related by

$u = u_x\mathbf{i} + u_y\mathbf{j}$

***The velocity u has components.** $u_x = u\cos\theta$, $u_y = u\sin\theta$.*

*First consider the Horizontal motion. **As the horizontal motion has no acceleration, the horizontal component of projectile velocity 'u_x' remains***

constant throughout the motion. The displacement of the projectile after any time 't' from the initial position

$x = u_x t = (u\cos\theta)$

*Now consider vertical motion****, the acceleration of the projectile is equal to the fall acceleration which is constant and always acted downward.***

$a = -gj$ *$a_y = -g$*

Hence the component of velocity at any time 't' is obtained by equation.

$V_y = u_y - gt$

Distance travelled by the projectile along vertical direction

$s = y$.

In vertical direction,

$u_y = u\sin\theta$ we get $V_y = u_y - gt$ *$V_y = u\sin\theta - gt$*

and also

$V_y^2 = u_y^2 - 2gt$ *$V_y^2 = (u\sin\theta)^2 - 2gy$*

These two equations are similar to the equations of motion of a body projected vertically upwards.

Also in the case of projectile, the velocity is initially directed upwards and its magnitude decreases to zero as the projectile reaches the maximum height vertically.

Displacement in vertical component $s = y = u_y t + \frac{1}{2}a_y t^2 = y$

By substituting $u_y = (u\sin\theta)$, and $a = -g$ we get

$(u\sin\theta)\, t + \frac{1}{2}(-g)\left(\frac{x}{u\cos\theta}\right) = y$, ***$y = u\sin\theta - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$***

From this we get:

$y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$ [Values g, θ, and u are constants]

Therefore, $y = Ax - Bx^2$ (where, $A = \tan\theta$, $B = \frac{g}{2u^2\cos^2\theta}$)

This equation represents parabola so that, the trajectory of the projectile is parabola.

The horizontal component velocity remains constant because there is no Acceleration in this direction.

MOTION IN APLANE

The vertical component velocity decreases gradually and finally becomes zero at the highest point.

TIME OF ASCENT:

The time is denoted by t_a.

At the maximum height, the vertical velocity is zero,

That is, $V_y = 0$

From the equation, $V_y = u\sin\theta - gt_a = 0$

We get, $t_a = \frac{g}{u\sin\theta}$

TIME OF FLIGHT (T)

The time of flight can be obtained by substituting

Y = 0, and t = T (time of flight) in equation

$y = (u\sin\theta)t - \frac{1}{2}gt^2$

i.e. $y = (u\sin\theta)t - \frac{1}{2}g(T)^2 = 0$

$T = \frac{2u\sin\theta}{g}$, On comparing equations, $t_a = \frac{u\sin\theta}{g}$ and $T = \frac{2u\sin\theta}{g}$

We get , $T = 2t_a$

MAXIMUM HEIGHT REACHED BY THE PROJECTILE

The Maximum Height reached by the projectile is determined by substituting time to reach maximum height $t = t_a$

$y = (u\sin\theta)t - \frac{1}{2}gt^2$ *$y = (u\sin\theta)t_a - \frac{1}{2}gt_a^2$*

Here , $t_a = \frac{u\sin\theta}{g}$

$H_{maximum} = (u\sin\theta)\frac{u\sin\theta}{g} - \frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$, $H_{maximum} = \frac{u^2\sin^2\theta}{2g}$

(When $\theta = 90^0$ i.e. the body projected vertically upwards)

$H_{maximum} = \frac{u^2}{2g}g$ (Since, $\sin 90^0 = 1$)

This is equal to the maximum height reached by a body projected vertically upwards.

MOTION IN APLANE

Horizontal range (R) of a projectile:

*The maximum horizontal distance travelled by the projectile from the point of projection during the time of flight is called **horizontal range**.*

Along horizontal direction:

Initial velocity $(u_x) = u\cos\theta$ *Acceleration* $(a_x) = 0$

Distance travelled (S)=R *Time taken (t) = T*

From $S = ut + \frac{1}{2}at^2$ $R = (u\cos\theta)T$

But $T = \frac{2u\sin\theta}{g}$ $\therefore R = \frac{(u\cos\theta)2u\sin\theta}{g} = \frac{2u2\cos\theta\sin\theta}{g}$

$\therefore R = \frac{u2\sin2\theta}{g}$ $(\because 2\sin\theta\cos\theta = \sin2\theta)$

Note : For a given value of projection velocity u, R is maximum when $2\boldsymbol{\theta} = 60^0$ *i.e* $\boldsymbol{\theta} = 45^0$

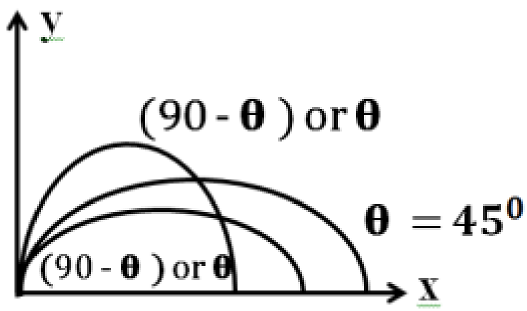
So maximum horizontal range is

$R_{max} = \frac{u^2}{g}$ $(\therefore \sin90^0 = 1)$

Note : for a given speed of projection 'u' the ranges are equal for angles

a) $\boldsymbol{\theta}$, $(90 - \boldsymbol{\theta})$

(b) $(45 + \boldsymbol{\alpha})$, $(45 - \boldsymbol{\alpha})$



Velocity of the projectile at any instant 't' :

The horizontal component of the projectile remains constant all the time (because acceleration due to gravity has no component) along the horizontal,

MOTION IN APLANE

∴ *Horizontal component of velocity after any time*

t is $V_x = u_x = u\cos\theta$

Vertical component of velocity after any times 't' is

$V_y = u_y - gt = u\sin\theta - gt$

$\overline{V} = (V_x\hat{i} + V_y\hat{j})$

$\overline{V} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

Then the magnitude of resultant velocity after time t is

$V = \sqrt{V_x{}^2 + V_y{}^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

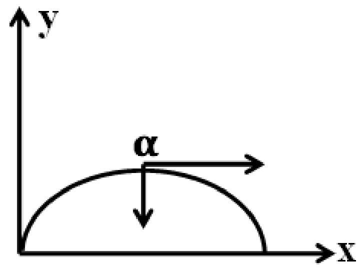
The velocity vector $\overline{V}$ *makes an angle* α *with the horizontal given by*

$\alpha = tan^{-1}\left(\frac{V_y}{V_x}\right)$ *at this instant*

At any vertical displacement velocity is

$\overline{V} = u\cos\theta\hat{i} + (\sqrt{u^2 sin^2\theta - 2gh})\hat{j}$

Note: The horizontal component of velocity remains constant along (since acceleration due to gravity has no component along the horizontal)



Example 1: *Two bodies are thrown with the same initial velocity at angles α and (90– α) to the horizon. What is the ratio of the maximum heights reached by the bodies?*

Solution :*Angles of two bodies thrown are = α, 90 – α*

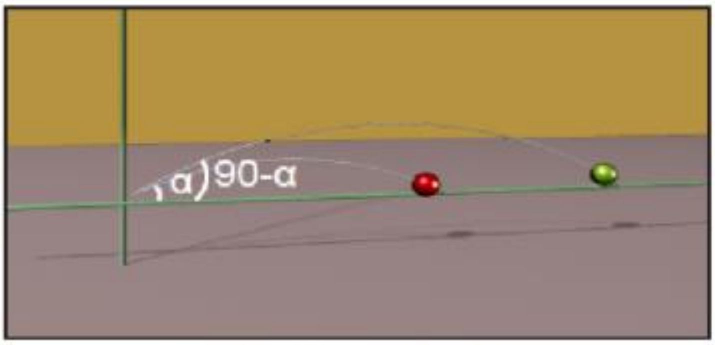
Ratio of the maximum heights = $\frac{H_1}{H_2}$

Horizontal range can also be termed as the horizontal distance travelled by the projectile during time of flight 'T'.

Therefore, $\frac{H_1}{H_2} = \frac{\frac{u^2 sin^2\alpha}{2g}}{\frac{u^2 sin^2(90-\alpha)}{2g}}$

$= \frac{sin^2\alpha}{cos^2\alpha}$

$\frac{H_1}{H_2} = tan^2\alpha$



Ex: Ratio of maximum heights

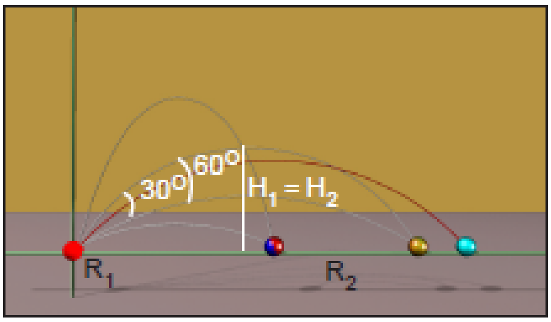
Example 2: *Two balls are projected from the same point in the direction inclined at 60^0 and 30^0 to the horizontal. If they attain the same height, what is the ratio of their velocities of projections? What is the ratio if they have same horizontal range?*

Solution : $\theta_1 = 60^0$ *and* $\theta_2 = 30^0$

Height is same i.e. $H_1 = H_2$

Calculate the ratio of the velocities of projection = ?

If the two bodies have same range i.e., $R_1 + R_2$



Ex: Horizontal range for same height

MOTION IN APLANE

HORIZONTAL PROJECTION FROM THE TOP OF A TOWER

In this module we discuss about horizontal projection of a body from the top of a tower.

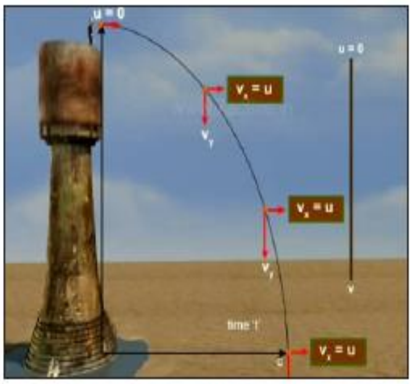
Consider a body thrown with an initial velocity 'u' from the top of a tower.

In the projection at every point the body possess two velocities, one is horizontal component of velocity and the other one is vertical component of velocity.

In this projection there is no acceleration along horizontal direction. Hence it moves with uniform velocity along horizontal direction.

In vertical direction it just falls like a freely falling body, because its initial vertical velocity is zero.

Let 't' be the time taken by the body to reach point 'c'.



Ex:Horizontal projection from the top of a tower

During this time x, y are horizontal and vertical displacements of the body along x and y direction respectively.

Horizontal displacement = horizontal velocity X time

(displacement = velocity X time)

i.e. x = ut

t = $\frac{X}{u}$***Equation (1)***

In vertical motion

u_y= 0, t = t, s = y, a = g,

S = ut + $\frac{1}{2}at^2$

Substitute above values in this formula, we get

Y = 0 + $\frac{1}{2}gt^2$

Y = $\frac{1}{2}g\,(x/u)^2$*[Since t = x/u]*

y = [$\frac{g}{2u^2}$*] X²*

let k =$\frac{g}{2u^2}$ *where 'k' is constant for a given velocity of projection.*

y = kx²

Above equation represents a parabola. Hence, we can conclude that trajectory of a body thrown horizontally from the top of a tower is a parabola.

Time of descent:

The time taken by the body to reach the ground from its maximum height is called its time of descent.

Along vertical direction:

Initial velocity (u_y)=0 *Acceleration (a_y)=+g*

Displacement (s)=h *Time taken(t)=t_d*

From $s = ut + \frac{1}{2}at^2$ $h = \frac{1}{2}gt_d{}^2$

$\therefore t_d = \sqrt{\frac{2h}{g}}$

Horizontal range (R):

The maximum horizontal distance travelled by a body during its time of flight.

Along horizontal direction:

Let Initial velocity (u_x)=u *Acceleration (a)=0*

Displacement (s)=R *Time taken(t)=t_d*

From $s = ut + \frac{1}{2}at2$

$R = u \times t_d$, $R = u \times \sqrt{\frac{2h}{g}}$

VELOCITY OF THE PROJECTILE AT ANY TIME ' t '

Let the body be at point 'p' after the time t, and its velocity be $v_x$and $v_y$along x and y directions.

The horizontal velocity remains constant throughout the motion.

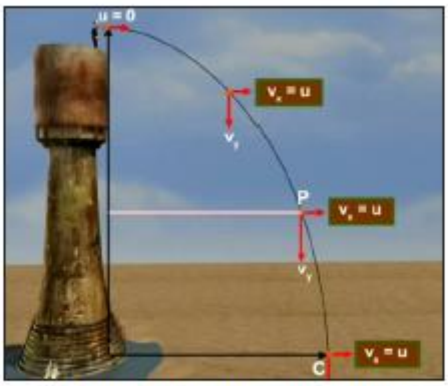
Hence v_x= u

The final velocity along Y – axis is

V_y= U_y+ gt and U_y= 0 as the body is thrown horizontally.

Therefore V_y= gt

After substituting $v_x$and $v_y$we get,



Ex: Horizontal projection from the top of a tower

MOTION IN APLANE

a) The magnitude of the velocity

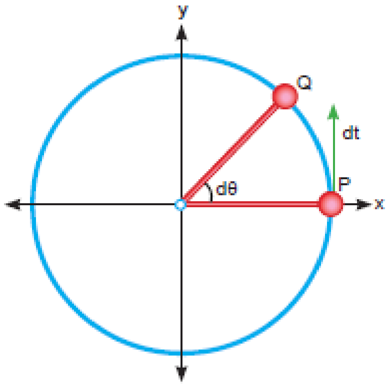
$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{u^2 + g^2t^2}$$

b) Direction of motion or angle made by the velocity vector with the horizontal is

$$\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{gt}{u}\right)$$

UNIFORM CIRCULAR MOTION :

*When a particle moves in a circular path such that its radius vector makes equal angular displacement in equal intervals of time then the particle is said to be in **uniform circular motion.***



Free body diagram

*When a particle moves on the circumference of a circle with **uniform angular velocity** or constant linear speed, then it is said to be in uniform circular motion.*

1. When the moon is revolving round the earth in a circular path, its motion is in uniform circular motion.

2. A satellite revolving round the earth in a circular orbit performs uniform circular motion. These are the examples of uniform circular motion,


Moon revolving round the earth


Satellite revolving round the earth

MOTION IN APLANE

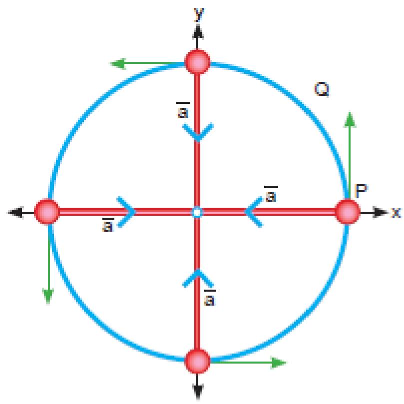
In uniform circular motion magnitude of linear velocity (speed), kinetic energy, angular velocity and angular momentum with respect to the centre of circle remains constant, but velocity changes due to change in direction. In uniform circular motion all real vectors like velocity, acceleration, force, momentum are constant in magnitude but continuously change in their direction.

In uniform circular motion, the angular velocity is constant and angular acceleration of the particle is zero. A particle moving with constant speed in a circle is not in equilibrium. (as net force is there which is acting as centripetal force).

CENTRIPETAL ACCELERATION

*When particle moves in a circular path with uniform speed, **the direction of velocity changes at every point on the circumference of the circle continuously. But the magnitude of the velocity remains constant.***

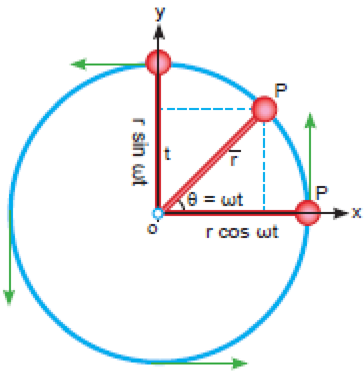
The change in the direction of velocity implies that there is some acceleration. This acceleration is called centripetal acceleration. It is directed towards the centre. Now, obtain an expression for the acceleration of a particle performing uniform circular motion.



Centripetal Acceleration

Derivation:

Consider a particle 'p' moving on the circumference of a circle of radius '$\bar{r}$' with constant angular velocity 'ω'. At any instant of time' t'. Let its position vector r be denoted by $\overline{OP}$. Taking its initial position along x-axis it rotates through an angle θ = ωt in a time interval 't'. Resolving $\bar{r}$in to x and y-components, we



Centripetal Acceleration

get x-component of r as (r cos ωt) $\bar{i}$ and y-component

of as (r sin ωt) $\bar{j}$

$\bar{r}$= (r cos ωt) $\bar{i}$+ (r sin ωt) $\bar{j}$

$\bar{r}$= r [(cos ωt) $\bar{i}$+ (sin ωt) $\bar{j}$] Equation (1)

To get the linear velocity of the particle the above expression is to be differentiated with respect to time.

$\bar{v}= \frac{d\bar{r}}{dt}$= rω[-(sin ωt) $\bar{i}$+ (cos ωt) $\bar{j}$] Equation (2)

From the above equation we observe that the velocity of the particle changes with time. So it will have linear acceleration.

Its linear acceleration can be obtained by differentiating the expression of the velocity of the particle with respect to time.

$\bar{a}= \frac{dv}{dt}$= rω^2[-(cos ωt) $\bar{i}$– (sin ωt) $\bar{j}$] = + rω^2[-[(cos ωt) $\bar{i}$+ (sin ωt) $\bar{j}$]

The magnitude of the linear acceleration is rω^2. The direction of the acceleration is given by –[(cosωt) $\bar{i}$+ (sinωt) $\bar{j}$] which is opposite to the direction of $\bar{r}$.

i.e the linear acceleration is directed along the radius towards the centre. So it is called centripetal acceleration. Since it acts along the radius and normal to velocity, it is also called radial (or) normal acceleration. Magnitude of centripetal acceleration

$a_c = r\omega^2 = v\omega = \frac{v^2}{r}$